Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name : Engineering Mathematics – I

Subject Code : 4TE01EMT2 **Branch: B.Tech (All)**

Semester : 1 **Date : 21/03/2018** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) If $y=(10)^{2x}$ then y_n equal to

(A) $2^n(10)^{2x}(\log 10)^n$ (B) $2^n(10)^{2x}(\log 2)^n$ (C) $2^n(10)^{2x}(\log 10)^{n+1}$

(D) none of these

b) If $y=\cosh 2x$ and if n is even, then y_n equal to

(A) $2^n\sinh 2x$ (B) $2^n\cosh 2x$ (C) $\cosh(2nx)$ (D) none of these

c) If $y=x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+.....$ then x equal to

(A) $y-\frac{y^2}{2!}+\frac{y^3}{3!}-\frac{y^4}{4!}+.....$ (B) $y-\frac{y^3}{3}+\frac{y^5}{5}-.....$ (C) $y-\frac{y^2}{2}+\frac{y^3}{3}-\frac{y^4}{4}+.....$

(D) none of these

d) The series $x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+.....$ represent expansion of

(A) $\cot^{-1}x$ (B) $\tan^{-1}x$ (C) $\sin^{-1}x$ (D) $\sin x$

e) $\lim_{x\to 0}\left(\frac{a^x+b^x}{2}\right)^{\frac{1}{x}}=$ _____

(A) $\log ab$ (B) $\sqrt{ab}$ (C) $\frac{1}{2}\log ab$ (D) none of these

f) $\lim_{x\to\infty}x^2e^{-x}=$ _____

(A) – 1 (B) 0 (C) 1 (D) none of these

g) If $P=r\tan\theta$, then $\frac{\partial P}{\partial r}$ is equal to

(A) $\sec^2\theta$ (B) $\tan\theta$ (C) $\tan\theta+r\sec^2\theta$ (D) $\frac{1}{2}\tan\theta$

h) If $u(x,y,z)=0$ then the value of $\frac{\partial x}{\partial y}\cdot\frac{\partial y}{\partial z}\cdot\frac{\partial z}{\partial x}$ is equal to

(A) 1 (B) – 1 (C) 0 (D) none of these

i) The value of $(1+i)^5\times(1-i)^5$ is

(A) -8 (B) $8i$ (C) 8 (D) 32

j) The polar form of the complex number $\frac{1+i}{1-i}$ is

(A) $\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$ (B) $\sin\frac{\pi}{2}+i\cos\frac{\pi}{2}$ (C) $\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}$

(D) $\sin\frac{\pi}{4}+i\cos\frac{\pi}{4}$

k) If $z=re^{i\theta}$, then $\left|e^{iz}\right|$ is equal to

(A) $e^{r\sin\theta}$ (B) $e^{-r\sin\theta}$ (C) $e^{-r\cos\theta}$ (D) $e^{r\cos\theta}$

l) If the system of equations $x-ky-z=0$, $kx-y-z=0$, $x+y-z=0$ has a non-zero solution, then value of k can be

(A) – 3 (B) $-\frac{1}{2}$ (C) 1 (D) 2

m) An eigenvalue of a square matrix A is $\lambda=0$. Then

(A) $|A|\neq 0$ (B) A is symmetric (C) A is singular (D) A is skew-symmetric

n) If every minor of order r of a matrix A is zero, then rank of A is

(A) greater than r (B) equal to r (C) less than or equal to r

(D) less than r

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) If $y=a\cos(\log x)+b\sin(\log x)$ then prove that **(5)**

$x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0$.

b) If $y=\frac{1}{x^2+a^2}$ then find y_n. **(5)**

c) Expand $f(x)=x^4-11x^3+43x^2-60x+14$ in powers of $(x-3)$. **(4)**

Q-3 **Attempt all questions** **(14)**

a) Expand $f(x)=\sec x$ in powers of x up to x^4 by Maclaurin's series. **(5)**

b) Prove that $\cos^{-1}\left[\tanh(\log x)\right]=\pi-2\left(x-\frac{x^3}{3}+\frac{x^5}{5}-.....\right)$. **(5)**

c) If $y=\sin^4 x$ then find y_n. **(4)**

Q-4 **Attempt all questions** **(14)**

a) Evaluate: $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x+d^x}{4}\right)^{\frac{1}{x}}$ **(5)**

b) If $u=f(r)$ and $r^2=x^2+y^2$ then prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r)$. **(5)**

c) If $z = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$ then show that $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = 0$. **(4)**

Q-5 **Attempt all questions** **(14)**

a) If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$ then show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$. **(5)**

b) Evaluate: $\lim_{x \to 1}\left(\frac{x}{x-1} - \frac{1}{\log x}\right)$ **(5)**

c) Evaluate: $\lim_{x \to \infty}\frac{1^2 + 2^2 + 3^2 + ... + x^2}{x^3}$ **(4)**

Q-6 **Attempt all questions** **(14)**

a) Solve the equation $z^3 = i(z-1)^3$. **(5)**

b) Using De Moivre's theorem prove the following: **(5)**
$\cos 5\theta = 5\cos\theta - 20\cos^3\theta + 16\cos^5\theta$

c) Check whether the following set of vectors is linearly dependent or linearly independent: **(4)**
$(1,2,-1,0),\ (1,3,1,2),\ (4,2,1,0),\ (6,1,0,1)$

Q-7 **Attempt all questions** **(14)**

a) Examine for consistency and if consistent solve them **(5)**
$x + 2y = 3,\ y - z = 2,\ x + y + z = 1$.

b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$. **(5)**

c) If $\sin(\alpha + i\beta) = x + iy$ then prove that $x^2 \operatorname{cosec}^2\alpha - y^2 \sec^2\alpha = 1$. **(4)**

Q-8 **Attempt all questions** **(14)**

a) If $x_r = \cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r}$ then prove that $\lim_{n \to \infty} x_1 x_2 x_3 ..... x_n = -1$. **(5)**

b) Reduce the matrix $A = \begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ to the normal form and find its rank. **(5)**

c) Discuss the continuity of the function **(4)**
$f(x,y) = \frac{x}{\sqrt{x^2 + y^2}}$ when $(x,y) \neq (0,0)$ and $f(x,y) = 2$ when $(x,y) = (0,0)$.